An Exposé of the Fallacy, " That it is just to tax Temporary Annuities at the same rate as Perpetual Annuities." By PETER HARDY, F.R.S., *one of the Vice-Presidents of the Institute of Actuaries, and Actuary to the Corporation of the London Assurance.*

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THE merits and demerits of the present mode of assessing the Property and Income Tax have been so frequently made the subject of popular discussion, and are in themselves questions of such national importance, that I feel no apology to be necessary for again bringing the subject under the notice of the Institute of Actuaries.

A valuable paper on the inequitable operation of this tax was, some time ago, read before the Institute by Mr. Jellicoe; and, more recently, an elaborate and important contribution on the same subject was brought by Mr. Farr under the notice of the Statistical Society of London, which paper has given rise to a lengthened and very interesting discussion. An exceedingly temperate and able pamphlet has also been recently published by Mr. John G. Hubbard, the Deputy Governor of the Bank of England, in which that gentleman has discussed, with very considerable ability, the views of Mr. Warburton, and has, in my opinion, very effectually exposed their absurdity.

Mr. Jellicoe's paper having been written previous to the examination of the scientific witnesses before the Committee of the House of Commons, its author could not be expected to contend with or reply to the very peculiar and startling theories propounded before that Committee by Mr. Warburton and Mr. Babbage. Mr. Farr, it is true, glanced in a casual manner at those and other fallacies; but his paper was intended to be an exposition of the true theory of direct taxation, and not a vehicle for opposing the particular views of individual supporters of the present system of taxation. Without attempting however, on this occasion, to venture on so lofty a flight as that of the three gentlemen who have preceded me, I shall be contented to address myself simply to an investigation, and, as I hope, to a refutation of the two distinct theories propounded by Mr. Warburton and Mr. Babbage, both of which theories tend to support an income and property tax, levied strictly in accordance with the existing impost.

I shall, however, in the present paper confine myself to Mr.

Warburton's theory alone, leaving for a future opportunity the discussion of Mr. Babbage's very peculiar views.

Mr. Warburton has, I believe, earned for himself an elevated name in the political and scientific world, and is, without question, abundantly endowed with qualifications for mathematical and financial research; to find, therefore, such a man advocating opinions so opposed (according to my view of the matter) to the common sense of the question, is a circumstance so remarkable, that I cannot but suspect that there is an important sophism lurking somewhere in the argument, a radical misunderstanding of some elementary definition; in short, it appears to me more than probable that every apparent difference between us may, after all, be traced to the different meanings which we respectively attach to the word "income." It is, therefore, most important to define clearly and distinctly the sense in which I employ this word, and the meaning which I attach to it.

Real income (as regarded by the year) is an annual profit—a return produced by the employment of capital. To constitute *real* income, it is absolutely essential that this return should be produced *by* and not *out of* capital; that is to say, it must be perpetually producible, *without destroying or in any degree impairing* the producing power of the capital itself. If in such annual returns any portions, however small, of the capital be included, and under a mistaken notion of their character, expended, it is obvious that such capital will be by so much lessened, that its producing power will be *pro tanto* diminished, and that, if the process be sufficiently continued, the whole capital will sooner or later be exhausted and disappear. It is, however, by no means a frequent circumstance that the nature of these returns of capital is misunderstood; on the contrary, whatever they may consist of, they are by the individual, as well as by the capitalist, carefully preserved and reinvested in some new form, so that on the exhaustion of the capital from the original investment, a new capital, equal to the former in amount, is created.

Real income, consequently, is not (as defined by some writers) the gross amount which comes into the receipt of any individual in the course of a year, but merely that portion of his receipts which he may safely expend without lessening his actual capital or weakening its productive power.

In all annual receipt of money it is, therefore, highly requisite to discriminate carefully between *real income* and *return of capital.* By way of illustrating this difference between incomes in which no

portion of repaid capital is included, and incomes in which certain portions are periodically returned, I will take the dividends on consolidated stock, the rent of land, interest on loans by way of mortgage, &c., as instances of *real income:* year after year, the stock, the land, the loan continue to produce to their respective owners *real incomes,* without exhausting or diminishing their productive power. But a temporary annuity, an annuity for a life or lives, and a professional income, are in like manner instances of *apparent incomes,* in which a portion of the capital originally invested is periodically repaid.

In the first two of these last examples it can be mathematically demonstrated (as will presently appear) that in each payment a portion of the original capital is in fact included.

In the case of a professional income, although mathematical proof cannot be adduced, yet common sense, and a little reflection, will show that with every beat of the pulse of life a portion of the strength and energy of the individual is wasted, and, consequently, a portion of the capital originally invested in education, or subsequently acquired by skill and industry, is expended, and that the last and closing scene of life witnesses the entire exhaustion of the original capital; and unless the repayments made in past years have been treasured with providence and fostered with care, the professional man will too often leave behind him no portion of that capital which has supported him and his family through life. In addition to the foregoing definitions of real income, I consider it essential to my argument to take up the following position, viz.:—

That an *income tax,* as such, can be levied on *income* only, and must in no case and in no degree affect the capital itself.

Mr. Warburton's theory is shortly this: "If the income tax is to be a temporary impost, it is *unjust* to tax temporary incomes at the same rate as perpetuities; but that if an income tax is to constitute a permanent burden on the means of the country, it is then perfectly equitable to tax a temporary income at the same rate, on the whole amount thereof, as a perpetuity;" and for this reason, "that whereas a *perpetual income* bears a *perpetual tax,* a *temporary income* bears only a *temporary tax.*" Now this I readily admit is a plausible theory to those who are unaccustomed to these and similar investigations, and subtle enough even to those who are. Mr. Hubbard has however most truly stated, in answer to Mr. Warburton's first proposition, that "no scheme of income tax, *unsound and unjust for a year,* will be just when the tax becomes perpetual."

My principal business in this paper being to endeavour to expose the fallacy of Mr. Warburton's entire proposition, I shall claim to be successful if I can show that in the adoption of this theory the tax is levied on any portion of the capital, no matter how small that portion may be. The best instance (because the one most easily investigated) which can be selected of a temporary income, is that particular Government or Parliamentary stock, known by the name of the Long Annuities. I shall confine my argument wholly to this class of temporary incomes, and shall base it on the following illustration:—

I lend a friend £1000, at 5 per cent. per annum interest. It is obvious that I may, in accordance with my previous definition, safely expend £50 annually, so long as the loan is continued, because such interest is *real income;* but it is equally obvious that I can expend no more without trenching on and diminishing my capital.

If my friend agree to repay the loan in the course of a couple of years by two equal instalments, I should receive at the termination of the first year the sum of £550, being, in point of fact, the interest on my £1000 for the year, and a repayment of one-half of the loan as agreed on. At the termination of the second year, I should in like manner receive the sum of £525, being, as before, a year's interest on the £500 left unpaid, together with the repayment of the balance of the loan. I would now venture to inquire, whether I should not exhibit utter ignorance of my true financial position, and display very bad economy, were I to consider my gross annual receipts in those two years, viz., £550 and £525 respectively, as income, and as such expend them? Palpable as such a blunder would appear to be, it is nevertheless the error virtually committed every day under the present system of taxation, and the justice of which is vindicated (as I shall presently proceed to show) by all who adopt Mr. Warburton's theory, and indeed by all who advocate the imposition of a tax on the actual receipts of each year.

It is unimportant what name may be given to this inequitable impost, but I would ask, If a tax were levied on the two gross annual receipts described in the foregoing illustration, whether such tax is not levied on the *gross capital* of the owner as well as on his *real income?* To this question there can be but one answer. Now extend the term of repayment, in the case of a similar loan, to *four years,* and we shall have in the four successive years the following payments, viz.—

	Principal. £.		Interest. £.	s.		£.	s.
At the end of the 1st year......	250	+	50	0	=	300	0
" " 2nd "	250	+	37	10	=	287	10
" " 3rd "	250	+	25	0	=	275	0
" " 4th "	250	+	12	10	=	262	10

I would again ask, If a tax be assessed on those four gross payments, viz., £300, £287. 10*s*., £275, and £262. 10*s*. respectively, would not, in such case, the tax have been levied on the gross capital as well as on the real income? Extend the term, if you will, to 50 or even to 100 years, and although you diminish the amount of injustice committed in each particular year, yet the total amount of injustice is the same; and in the end the tax is levied on the gross capital, as well as on the accruing income.

It will be now my business to show that a long annuity is nothing else than a loan contracted by Government, and originally agreed to be repaid by 60 annual instalments,* each dividend containing within itself a portion of the original capital repaid. It is quite true that the payments which include these instalments are constant in amount, viz., £1 in each year, while the payments of the loan in the illustrations already given vary in amount in each year, the sum repaid being constant. It can, however, be readily shown, that there is no substantial difference between the two, and that in each case a tax levied on the apparent income of each year is levied not only on the real income or interest, but on the entire capital as well. In the case of a loan repayable by equal instalments, let V represent the capital put out for n years at i per £1 per annum interest, then $\frac{V}{n}$ will be the amount of capital annually repaid, and the successive payments will be

$$\left(\frac{V}{n}+Vi\right)+\left(\frac{V}{n}+\frac{\overline{n-1}.Vi}{n}\right)+\left(\frac{V}{n}+\frac{\overline{n-2}.Vi}{n}\right)+, \text{ \&c. to } n \text{ terms.}$$

Now one portion of each term, viz., $\frac{V}{n}$, is obviously constant in each term, while the other portions of each term, viz.,

$$\left(\frac{n.Vi}{n}\right),\left(\frac{\overline{n-1}.Vi}{n}\right),\left(\frac{\overline{n-2}.Vi}{n}\right), \text{ \&c.,}$$

* We are all well aware that the long annuity is payable half-yearly, but it is unnecessary to complicate our argument by dealing with it, excepting as an annual payment.

as obviously annually decrease; each payment, being an entire term, must consequently decrease too, and the sum of the n terms will, by a very easy summation, be found equal to

$$V+\frac{\overline{n+1}.Vi}{2}:$$

thus the whole capital (V) originally put out to interest is repaid within the term, together with $\frac{\overline{n+1}Vi}{2}$ total interest, and a tax levied on each full term of this series is obviously levied on V, as well as on $\frac{\overline{n+1}.Vi}{2}$.

Now if these decreasing terms, in which the sum repaid in each year is constant throughout the series, be equated, and the terms themselves made constant, then the instalments of capital repaid in each year, and the interest thereon, will both vary in each term—the one increasing, and the other decreasing, throughout the whole series. Putting V_n, or its equivalent,

$$\frac{1-\frac{1}{(1+i)^n}}{i},$$

equal to the value of an annuity of a constant £1 per annum for n years, which is also to be equal to the sum invested, and putting $V_{(n-1)}$, or its equivalent,

$$\frac{1-\frac{1}{(1+i)^{n-1}}}{i},$$

equal to the value of a similar annuity for $\overline{n-1}$. years, and so on, we shall obtain the three following series, viz.—

$$1+1+1+1+1, \text{ \&c. to } n \text{ terms},$$

for the actual full terms of the series, or total payments made in each year;

$$V_n i+V_{n-1}i+V_{n-2}i+V_{n-3}i+, \text{ \&c. to } n \text{ terms},$$

for the amounts of interest paid in each successive year; and consequently

$$(1-V_n i)+(1-V_{n-1}i)+(1-V_{n-2}i)+(1-V_{n-3}i)+, \text{ \&c. to } n \text{ terms},$$

for the amounts of capital repaid in each successive year.

Now the sum of the second series is manifestly equal to

$$i\,(V_n+V_{n-1}+V_{n-2}+V_{n-3}+,\ \&c.)=i\left(\frac{1}{i}\left(n-\frac{1-\frac{i}{(1+i)^n}}{i}\right)\right)$$

$$=n-\frac{1-\frac{1}{(1+i)^n}}{i};$$

and consequently the total capital repaid within the entire term will be the sum of the third series, which is manifestly equal to

$$\frac{1-\frac{1}{(1+i)^n}}{i}=V_n=\text{ to the sum originally invested:}$$

consequently, a tax which is levied on each £1 throughout the whole series is not only levied on the interest, but is also levied on V_n (the capital) as well.

It is hardly necessary that I should explain that this last series is neither more nor less than a long annuity; and I believe that I may fairly claim to have satisfactorily made out, that in the case of every temporary annuity where the tax is levied on the dividend received in each year, it is in truth levied on the capital as well as on the interest.

Looking at the strictly mathematical character of the foregoing reasoning, it would appear that Mr. Warburton's peculiar view of the justice of taxing each full payment of a temporary annuity must of necessity arise from the circumstance of his having assigned a meaning to the word "income" quite different to that which is the true one. If he regard the periodical receipt of money, whether real or apparent income, as an avowal on the part of the receiver of an intention to expend it, or if all capital repaid by way of periodical instalment were actually annihilated on being so repaid, I could then comprehend the justness of his reasoning and the soundness of his theory, because in such case the temporary tax on the larger temporary dividend would be exactly equivalent to the perpetual tax on the smaller but perpetual dividend; but as such in truth is not the case—as capital so repaid by periodical instalment is not and cannot be treated by the receivers thereof, whether private individuals or public capitalists, as income—and as such capital, so far from being annihilated, reappears in some other form of investment, and constitutes a portion of the general property of the country—so any attempt to argue that it is annually expended is unfair, and every attempt to tax it is unquestionably unjust.

The flimsy argument that capitalists have chosen to invest their money in this particular sort of security, called temporary annuity, because it is more convenient, or perhaps because it offers greater apparent advantages to them than other contemporary investments, and that they ought therefore to be made to pay for such special considerations by a tax on their capital, is so manifestly an absurdity that it scarcely deserves a serious denial. Another argument which has been brought forward is, that the present holders of long annuities have received compensation for the heavier rate of taxation with which they are burdened, in the price which they paid for the original purchase of their stock. This is untrue; and even if it were true, it would in no way justify the original imposition nor the present maintenance of such a tax, because such compensation could only have been obtained at the cost of other individuals equally entitled with the present holders to the protection of the State, inasmuch as they must have made their purchases at a lower comparative rate than that at which the same stock was originally purchased by the seller. If Government, indeed, granted such annuities at the present time, and in doing so sold them to buyers at rates sufficiently low to protect them from loss by the operation of the income tax, in such case doubtless the holders would be compensated, and they would have no real cause for murmur or complaint. But it is not so; and even if it were, it is evident that the loss of the Government would in each case be precisely the same as the gain of their customers.

In a leading article on the subject of the income tax, the *Morning Chronicle* very recently endeavoured to find an absurdity in my argument in behalf of the present holders of long annuity, on the ground that I had asserted, that if any compensation were really made by a reduction in the cost of the annuity, it was the seller and not the present holder that suffered. Surely, however, the *Chronicle* will admit that the holder of to-day may be the seller of to-morrow, and that he is fully entitled to claim protection against a wrong with which he is threatened, as against one which is actually perpetrated.

I think it will be admitted that if I have made out a case of injustice as regards the long annuities, that such reasoning will in like manner maintain the injustice of the operation of the tax on every other description of temporary annuity, whether for very long or for very short periods, whether certain or uncertain, whether for a life or lives.

It would not be more difficult to show that Mr. Warburton's

theory is equally untenable in the case of professional incomes; but this is so large and important a question, and will be so fully investigated in my proposed reply to Mr. Babbage, that I shall not enter on the inquiry on the present occasion.

The question of annuities purchased by individuals on their own lives admits of a more elaborate discussion. It has been argued with some speciousness, that if it could be made apparent that a life annuity when purchased was intended to be wholly expended, Government would be then entitled to deduct the tax from the full annuity, because each payment of such annuity containing as it does a portion of capital which by expenditure is supposed to be destroyed, and the State being admitted to be entitled to a *perpetual tax on the real income of all capital,* no member of the community has the right to withdraw or annihilate as it were his individual portion of the aggregate wealth of the country, merely to increase his own present means of enjoyment, or to afford himself a greater abundance of comforts and luxuries. But this is manifestly a fallacy. No capital can in reality be expended so as to be destroyed—no portion of capital can be withdrawn from the national wealth, unless indeed its proprietor were to throw it to the bottom of the sea, or to dig a hole and bury it in his garden. The capital of the annuitant, if it be expended by him, passes into the possession of others, and is in their hands and under their ownership, producing income and bearing its fair share of the taxation of the country; but besides this, we have no right to presume that any individual who converts a perpetual into a life income acts from any other motives than those which influence the capitalist and the financier, who remove their property from one sort of investment to another. If the justice of taxing an annuity, merely because it is supposed to be actually expended, were to be once admitted, it would not be very difficult to justify the imposition of a tax on all expenditure, whether within or beyond the means of the individual—whether on the expenditure of the provident father who, beyond his income, devotes some portion of his capital to the education of his children, or whether on the idle spendthrift of middle life who incurs beyond his means debts hopeless of their future liquidation, or on the gartered noble who expends on the turf or over the gaming table more than the rental of his patrimonial acres.

I shall now bring this paper to a close. Much has been said and much has been written on the subject. The objectors to the present system of taxation have been most unfairly charged with

having endeavoured by popular clamour to overthrow a just and necessary tax, and with having argued against it from individual and selfish motives. Neither of these accusations is true. The supporters of the tax, on the other hand, have resisted every attempt to inquire into its injustice, and have brought forward in its favour arguments as transparent as they are feeble. This is not the way in which a great national question ought to be met. Men's minds have expanded within the last fifty years in a ratio previously unknown; and the present generation look for something more than empty declamation, or appeals to a former statesman and his measures. They cannot accept as unerring truths the exploded dogmas of Mr. Pitt, merely because they have been hallowed by the lapse of half a century, or because they have received the sanction of preceding and perhaps subservient senates.

On the Reliability of Data, when tested by the conclusions to which they lead. By EDWIN JAMES FARREN, *one of the Vice-Presidents of the Institute of Actuaries.*

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IT is a well-known feature in the advancement of learning, that the more knowledge becomes prevalent, the less positive do opinions become. Not only in theology does the staggering demand of "What is truth?" assail us, but we meet with the same impregnable question in all the pursuits of literature and science, obliging us to admit that the truth of yesterday is not always the truth of to-day, and that to-morrow may still be about to offer us some third form.

The older class of writers felt no such difficulty. They conceived they were in possession of certain abstract truths or data that might be considered as fundamental axioms, and that by these axioms might all conclusions whatsoever be tried. If the conclusions were incredible, it was the logic that was to be impeached, not the axioms; for it was taken as the reigning axiom of all, that axioms themselves could not be misconceived or defective. The more general lover of research owes it to mathematicians, that this mechanical form of procedure, that this exaltation of one class of assumptions as tyrants over all others, has become disregarded; for it was by labourers in the exact sciences that it was first fully shown, that starting from